

41 $y=0$ $y=2$ \perp y axis

$x = \sqrt{5} y^2$

$\int_c^d A(y) dy$

$\int_0^2 \pi r^2 dy$

$\int_0^2 \pi \left(\frac{\sqrt{5} y^2}{2}\right)^2 dy = 8\pi$

Dec 15-9:12 AM

7.4 Length of a smooth curve

Approximate the length of the curve $y = \sin(x)$ from $x = 0$ to $x = 2\pi$

$7.6404 = \int_0^{2\pi} \sqrt{1 + (\cos x)^2} dx$

$\Delta s_i^2 = \Delta x_i^2 + \Delta y_i^2$

$\Delta s_i = \sqrt{\Delta x_i^2 + \Delta y_i^2}$

$S = \lim_{\Delta x \rightarrow 0} \sum \sqrt{\Delta x^2 + \Delta y^2}$

$S = \lim_{\Delta x \rightarrow 0} \sum \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2} \Delta x$

$S = \lim_{\Delta x \rightarrow 0} \sum \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \Delta x$

$S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

Dec 17-5:53 PM

Definition of arclength

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

use if $y = f(x)$
 $\frac{dy}{dx}$ must exist on (a, b)

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

use with $x = g(y)$
 $\frac{dx}{dy}$ must exist on (c, d)

Dec 17-6:43 PM

Find the exact length of the curve $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1, 0 \leq x \leq 1$

$\frac{dy}{dx} = \frac{4\sqrt{2}}{3} \cdot \frac{3}{2} x^{\frac{1}{2}} = 2\sqrt{2} \sqrt{x}$

$\left(\frac{dy}{dx}\right)^2 = (2\sqrt{2} \sqrt{x})^2 = 4 \cdot 2 \cdot x = 8x$

$L = \int_0^1 \sqrt{1 + 8x} dx = \frac{1}{8} \cdot \frac{2}{3} (1 + 8x)^{3/2} \Big|_0^1$

$= \frac{1}{12} (9^{3/2} - 1^{3/2}) = \frac{1}{12} (27 - 1) = \frac{26}{12} = \frac{13}{6}$

Dec 17-6:49 PM

A vertical tangent

Find the length of the curve $y = \sqrt[3]{x}$ between $(-8, -2)$ and $(8, 2)$

$$x = y^3$$

$$\frac{dx}{dy} = 3y^2 \quad \int_{-2}^2 \sqrt{1 + (3y^2)^2} dy = 17.261$$

$$\frac{dy}{dx} = \frac{1}{3}x^{-2/3}$$

$$= \frac{1}{3}x^{1/3} \text{ dne at } x=0$$

$$\int_{-8}^8 \sqrt{1 + \left(\frac{1}{3}x^{1/3}\right)^2} dx$$

gues. accuracy

Dec 17-6:59 PM

A cusp

Find the length of the curve $y = x^2 - 4|x| - x$ from $x = -4$ to $x = 4$

red: $x < 0 \quad |x| = -x$

$$x = -3 \quad |x| = -x$$

$$|-3| = -(-3)$$

$$y = x^2 - 4(-x) - x$$

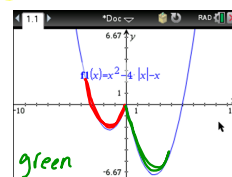
$$y = x^2 + 3x$$

$$\frac{dy}{dx} = 2x + 3$$

$$\int_{-4}^0 \sqrt{1 + (2x+3)^2} dx = 9.7782$$

$$\text{total} = 9.7782 + 9.7782$$

$$= 19.5564$$



green: $x > 0 \quad |x| = x$

$$y = x^2 - 4x - x$$

$$y = x^2 - 5x$$

$$\frac{dy}{dx} = 2x - 5$$

$$\int_0^4 \sqrt{1 + (2x-5)^2} dx = 9.7782$$

Dec 17-7:16 PM

Jan 3-12:44 PM